VOLTAGE-CURRENT CHARACTERISTICS OF A CYLINDRICAL LANGMUIR PROBE IN A SLOWLY MOVING PLASMA

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Theoretical determination of the voltage-current characteristics of a Langmuir probe in a dense plasma has proven to be quite a complex problem. It has been successfully solved only for a spherical probe in a nonmoving plasma [1]. Numerical calculations in [1] yielded voltage-current characteristics over a wide range of both the probe potential and the ratio $\alpha = \lambda_D/R$, where λ_D is the Debye radius and R is the probe radius. An approximate, asymptotic solution has also been achieved for $\alpha \ll 1$ and moderate probe potential [2].

Theoretical analysis of the cylindrical probe is more complex. There are two approaches to the problem. In the first, a finite-length probe is considered to be a section of an infinitely long probe. Here the solution for a nonmoving plasma, analogous to those of [1, 2], cannot be obtained because it is impossible to satisfy the boundary conditions at infinity. In the second approach, the cylindrical probe is approximated by an ellipsoid of revolution. This approach was applied to a nonmoving plasma in, for instance, [3], but in that work only the saturation current was obtained due to the complexity of the problem.

The saturation current method [4] is presently the most commonly used approximate method for moving plasmas, where determination of the voltage-current characteristics is more difficult. It allows derivation of practical relations for diagnostics of various plasma flows. An important regime of plasma motion is low Reynolds number flow, seen, for example, in probe measurements of laboratory flames. The ion saturation current was obtained in [5] in a slowly moving plasma with electrical Reynolds number $Re_e \ll 1$, for a cylindrical probe approximated by an ellipsoid of revolution. The saturation current at a section of an infinitely long cylindrical probe was calculated in [6] over the range $1 \leqslant Re_e \leqslant 15$.

To determine the charged-particle density from experimentally obtained ion saturation current characteristics, the question arises of how to select the current characteristic for which the current is equal to the theoretical saturation current. In the present work, following [2], we determine the voltage-current characteristics of a cylindrical probe in a plasma with $Re_e \ll 1$. This allows the question to be answered not only for $Re_e \ll 1$, but (approximately) also for $Re_e \sim 1$, typical of probe measurements in flames.

1. Consider a collisional plasma flow near an infinitely long cylindrical conductor (probe) whose axis of symmetry is perpendicular to the velocity U_∞ of the incident flow. The flow takes place with $Re_e \ll 1$. The plasma consists of neutral particles, positively charged ions, and negatively charged particles (electrons or ions); the neutral density is much greater than that of the charged particles, so that the plasma is weakly ionized. The charged-particle density is such that $\alpha \ll 1$. The ion and neutral temperatures are assumed equal and constant throughout the flowfield. When negative particle transport is carried out by electrons, the temperature T_- of negatively charged particles may differ from the positive-charge temperature T_+. However, we assume that the ratio $\tau \equiv T_+/T_-$ remains constant. Chemical reactions in the flow are considered to be frozen.

Negative ions are brought into consideration here because of their presence in combustion-formed plasmas [7], where under certain conditions they play a substantial role [8]. Two limiting cases are considered below, in which negative charge is transported only by electrons or only by ions.

Under the present assumptions, plasma probe operation is described by the following equations [9], in dimensionless form:

$$\tau\,\mathrm{Re}_e(\mathbf{u}\nabla n_+) - \nabla(\tau\nabla n_+ - n_+\nabla\psi) = 0; \tag{1.1}$$

$$\beta\,\mathrm{Re}_e(u\nabla n_-) - \nabla(\nabla n_- + n_-\nabla\psi) = 0; \tag{1.2}$$

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$$\alpha^2\nabla^2\psi = n_+ - n_-. \tag{1.3}$$

Here $Re_e = U_\infty R/D_+$; u is the neutral gas velocity nondimensionalized by the incident flow velocity; n_+ and n_- are the positive and negative charged-particle densities, nondimensionalized by the density of the incident flow; ψ is the dimensionless potential obtained by normalizing the dimensional potential ψ : $\psi = -e\varphi/kT_-$ (here e is the electron charge and k is Boltzmann's constant); and $\beta = D_+/D_-$ is the ratio of the positive and negative charged-particle diffusion coefficients. When the negative particles are only electrons, $\beta \ll 1$, and when only ions, β is of order unity (we will set $\beta = 1$).

We will take the velocity field of the neutral gas to be given; because of the assumed weak ionization, it does not depend on the presence of charged species. The boundary conditions for (1.1)-(1.3) are as follows: at the probe surface (r = 1),

$$n_+ = n_- = 0, \quad \psi = \psi_p ; \tag{1.4}$$

where ψ_p is prescribed; and far from the surface ($r \to \infty$),

$$n_+ \to 1, \quad n_- \to 1, \quad \psi \to 0. \tag{1.5}$$

If we neglect the convective terms in (1.1) and (1.2) because of the low Re_e, transform to polar coordinates (r, θ), and do not consider the θ-dependence of the desired functions, then (1.1) and (1.2) may be integrated:

$$\tau\frac{dn_+}{dr} - n_+\frac{d\psi}{dr} = \frac{\tau I_+}{r}; \tag{1.6}$$

$$\frac{dn_-}{dr} + n_-\frac{d\psi}{dr} = \frac{I_-}{r} \tag{1.7}$$

where the constants of integration I_+ and I_- represent the dimensionless currents of positive and negative charged particles onto the probe. Equations (1.6) and (1.7) are exact for a nonmoving plasma ($Re_e = 0$).

It is well known that for $\alpha \ll 1$, it follows from (1.3) that throughout the region of space outside the thin space charge layer adjacent to the probe surface, the plasma is quasineutral ($n_+ \approx n_- = n$). Adding (1.6) to (1.7) and integrating, the charge density in the quasineutral region becomes

$$n = A + \frac{\tau I_+ + I_-}{1+\tau}\ln r \tag{1.8}$$

where A is a constant of integration.

Even if one of the currents I_+ or I_- differs from zero, it is clear from (1.8) that as $r \to \infty$ the density n grows without limit. The boundary condition (1.5) thus cannot be satisfied. Therefore, no solution of (1.1)-(1.5) exists for $Re_e = 0$, although one can be found for $Re_e \ll 1$.

2. The given problem is similar to that of two-dimensional flow of an unbounded fluid over a cylinder for low Reynolds number [10]. Even for very small Re_e, the convective terms of (1.1) and (1.2) cannot be fully neglected. They may be neglected close to the probe, but they must be retained at large distances where, irrespective of the magnitude of Re_e, the convective terms become of the same order as the diffusive terms.

The solution (1.8) is approximately valid in the quasineutral region near the probe, and is the primary term in the interior asymptotic expansion of the charged particle density in Re_e. We now obtain a solution which is valid far from the probe.

Summing (1.1) and (1.2) and using quasineutrality,

$$\nabla^2 n - 2\varkappa(\mathbf{u}\nabla n) = 0, \tag{2.1}$$

where $\varkappa = (\tau+\beta)Re_e/2(1+\tau)$.

Looking at (2.1), it is easy to see that the condition $Re_e \ll 1$ in the problem statement may be replaced by $Re_e \ll (1 + \tau)/(\tau + \beta)$. The second term of (2.1) is then negligibly close to the probe. The conditions are identical in the two limiting cases of negatively charged particles being ions only ($\beta = 1$) and electrons only ($\beta \ll 1$), but in the second case, the given theoretical analysis is applied for somewhat higher values of Re_e than in the first.

Far from the probe, the second terms of (2.1) cannot be neglected; moreover, the flow velocity is close to the undisturbed value: $u_r = \cos\theta$ and $u_\theta = -\sin\theta$, where u_r and u_θ are the velocity components. The solution of (2.1) which satisfies condition (1.5) is

$$n = 1 + \exp(\varkappa r\cos\theta)\sum_{m=0}^{\infty} B_m K_m(\varkappa r)\cos m\theta. \tag{2.2}$$

Here K_m is the m-th order modified Bessel function of the second kind, and B_m are constants of integration.

For small values of $\varkappa$ at a fixed distance r, taking the first term of the series expansion of $K_0(\varkappa r)$, we obtain the first term of the interior expansion of the exterior solution:

$$n = 1 - B_0(C + \ln(\varkappa r/2)) \tag{2.3}$$

where C = 0.5772... is the Euler constant. The constants B_1, B_2,... are set equal to zero so that the solution obtained using (2.3) does not depend on the angle θ and may be asymptotically joined with the interior solution given by (1.8).

In accordance with the principle of asymptotic joining [10], the constants A and B_0 in (1.8) and (2.3) take the form

$$A = 1 + \frac{\tau I_+ + I_-}{1+\tau}\left(C + \ln\frac{\varkappa}{2}\right), \quad B_0 = -\frac{\tau I_+ + I_-}{1+\tau}$$

and the quasineutral density near the probe is given by the expression

$$n = 1 + \frac{\tau I_+ + I_-}{1+\tau}\left(C + \ln\frac{\varkappa}{2} + \ln r\right). \tag{2.4}$$

From (2.4), setting n = 0 at r = 1 and I_+ or I_- to zero, we obtain for the saturation currents (see [5])

$$I_+^* = -(1 + 1/\tau)/(C + \ln(\varkappa/2)), \quad I_-^* = \tau I_+^*. \tag{2.5}$$

Integrating (1.6), subject to (2.4), the primary term of the interior asymptotic expansion of the potential in the quasineutral region is

$$\psi = -\frac{\tau(I_+ - I_-)}{\tau I_+ + I_-}\ln\left[1 + \frac{\tau I_+ + I_-}{1+\tau}\left(C + \ln\frac{\varkappa}{2} + \ln r\right)\right] + c_1. \tag{2.6}$$

The constant c_1 must be determined by joining the potential obtained from (2.6) with the solution for the potential in the exterior region. We show that c_1 = 0. Multiplying (1.1) by β and (1.2) by τ, and subtracting one from the other, we have

$$\nabla[(\beta+\tau)n\nabla\psi + \tau(1-\beta)\nabla n] = 0, \tag{2.7}$$

which is valid through the quasineutral region and for any Re_e. The solution of (2.7) is

$$\psi = -\frac{\tau(1-\beta)}{\tau+\beta}\ln n + \psi_1, \tag{2.8}$$

where ψ_1 satisfies

$$\nabla(n\nabla\psi_1) = 0 \tag{2.9}$$

and $\psi_1 \to 0$ as $r \to \infty$.

Let us find the solution of (2.9) in the interior region. Taking into consideration that the possibility of asymptotic joining of the potential component ψ_1 in the interior region cannot depend on the angle θ, and using (2.4), we have

$$\psi_1 = c_2\frac{1+\tau}{\tau I_+ + I_-}\ln\left[1 + \frac{\tau I_+ + I_-}{1+\tau}\left(C + \ln\frac{\varkappa}{2} + \ln r\right)\right] \tag{2.10}$$

where c_2 is a constant of integration.

Because (2.7) is valid throughout the quasineutral region, Eq. (2.8), in which the density n is determined from (2.4) while ψ_1 is given by (2.10), may be considered as the first term of the exterior asymptotic expansion of the potential in the interior region. In accordance with the principle of asymptotic joining, in (2.10), $c_2 = -\tau(\beta I_+ - I_-)/(\tau + \beta)$, and in (2.6), c_1 = 0.

Note that a similar problem was considered in [12]. The difference from the present work is that in [12], a conventional reference electrode was used. The reference electrode was a large-radius cylinder, penetrable by the plasma, at which the potential was zero. The resulting computed voltage-current characteristics depended on the distance between the reference electrode and the probe surface. However, removal of the electrode to infinity leads in [12] to an infinite probe potential. Clearly, in [12], an erroneous value of c_1 was assumed in (2.6).

3. Let us make the substitution $\xi = \ln r$. Equation (2.6) shows that as $\psi \to \infty$, $\xi \to \xi_s$, where

$$\xi_s = -(1+\tau)/(\tau I_+ + I_-) - \ln(\varkappa/2) - C.$$

Consequently, the quasineutral solutions (2.4) and (2.6) are valid only for $\xi > \xi_s$. To analyze the space charge layer for $\xi \lesssim \xi_s$, we make use of the method suggested in [2].

During $E = d\psi/d\xi$, Eqs. (1.3), (1.6), and (1.7) may be combined into one equation for E. Making the transformation

$$\zeta = a\alpha^{-2/3}(\xi - \xi_s), \quad E(\xi) = a\alpha^{-2/3}F(\zeta)$$

where $a = [(\tau I_+ + I_-)\exp(2\xi_s)/\tau]^{1/3}$, and neglecting terms of order $\alpha^{2/3}$, we arrive at an equation for F, which may be integrated to obtain the equation for the space charge layer [2]:

$$F'' = (1/\tau - 1)FF' + (1/2\tau)F^3 + \zeta F + \lambda \tag{3.1}$$
$$(\lambda = \tau(I_+ - I_-)/(\tau I_+ + I_-)).$$

The solution of (3.1) must approach the quasineutral solution (2.6) as $\zeta \to \infty$. Using the new variables, this takes the form

$$F \to -\lambda/\zeta \quad \text{as} \quad \zeta \to \infty. \tag{3.2}$$

At the probe surface [2],

$$F'_p = 0 \quad \text{for} \quad \zeta_p = -F_p^2/2\tau, \tag{3.3}$$

where the subscript p denotes the values of a variable at the surface.

The solution of (3.1)-(3.3) proceeds as follows. For a given value of λ, a point (ζ_p, F_p) on the probe surface is selected, such that integration using (2.3) also satisfies (3.2). When the value of ζ_p has been found, I_+ and I_- may be determined using the relation $\zeta_p = -a\alpha^{-2/3}\xi_s$ and the expression for λ. The probe potential is then calculated from

$$\psi_p = \int_{\zeta_p}^{\zeta_*} F\,d\zeta + \psi(\zeta_*),$$

where ζ_* is the point at which the integration is joined with the quasineutral solution, and the potential $\psi(\zeta_*)$ is obtained from (2.6) for the quasineutral region.

The solution of (3.1) was performed numerically by converting it to a system of first-order differential equations and then to difference equations according to [13]. For various values of λ, the corresponding values of ζ_p were obtained in [2] such that condition (3.2) was satisfied. However, in the present work, the values given in [2] are used only as guidelines.

Depending on ζ_p, we can identify two forms of the integral curves. If the magnitude of ζ_p is somewhat smaller than that which satisfies (3.2), then the integral curve approaches the curve describing the quasineutral solution, and later intersects it. If somewhat larger, then the difference between the two solutions gradually decreases in magnitude, attains a minimum, and then begins to increase. We may achieve an increasingly accurate representation of ζ_p so that the minimum does not exceed a small quantity (10^{-4}) specified beforehand. In that case we regard the value of ζ_p to be known, and the value at the minimum is ζ_*.

The value of ζ_p is searched for during computation. The resulting values, for given λ, obtained here are rather lower in magnitude than the corresponding data in [2]. This is apparently due to the more up-to-date computational method used here.

Note that the method developed in [2] allows the characteristics to be obtained only for moderate values of the probe potential. For high probe potentials, the space charge layer has a complex structure; this case requires special attention [14].

4. The calculated voltage-current characteristics are shown in Figs. 1-3. For $\tau = 1$, they are symmetrical with respect to the ordinate (Figs. 1, 2). The saturation current levels are shown by dashed curves. In Fig. 3, $Re_e = 0.2$ and $\tau = 0.5$.

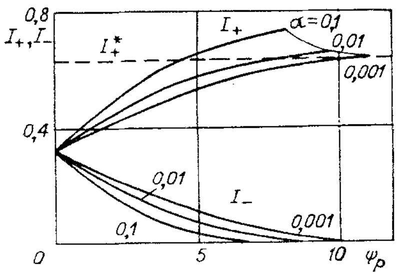

Fig. 1

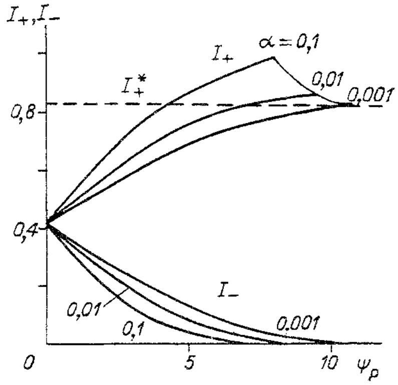

Fig. 2

It may be seen from the figures that when the ion current attains saturation ($I_- \to 0$), its value is extremely close to the theoretical value (2.5) only for sufficiently small α ($\alpha = 10^{-3}$). Ion-current saturation then becomes independent of τ for a potential $\psi_p \simeq 10$. For large α (for example, for $\alpha = 0.1$) the probe current, upon saturation, exceeds the theoretical value by about 25%. The value $\alpha = 0.1$ occurs quite often in probe measurements. Consequently, use of the theoretical value of the saturation current to determine the density, as in [6], can lead to overestimation of the density due to the finite α.

Note that the values $Re_e = 0.2$ and 0.4, given in Figs. 1 and 2, corresponding to the case in which transport of negative charge is carried out by electrons. If the negative particles are ions, then $Re_e = 0.1$ and 0.2 (Figs. 1 and 2). Thus, for a given value of Re_e, the dimensionless probe current in the negative-ion case increases. Comparing Figs. 1 and 2, we may estimate that the increase in ion current due to the presence of plasma negative ions is about 30%.

5. Compare the voltage-current characteristics obtained with those of a cylindrical probe, 0.5 mm in diameter, taken in a sodium-seeded air-acetylene flame (Fig. 4). Because Re_e, considered in the previous sections, is difficult to determine experimentally, the comparison is only qualitative. The gas flow velocity in the flame was 4.4 ± 0.5 m/sec, the temperature was 2370 ± 10 K, and the ion diffusion coefficient was $D_+ = 5.2$ cm^2/sec [15]. From these data, $Re_e = 2.1$.

In terms of the dimensionless charged-particle currents, the dimensional probe currents $J_\pm$ are

$$J_\pm = 2\pi e N_\infty D_\pm L I_\pm, \tag{5.1}$$

where N_∞ is the charge density far from the probe and L is the probe length. The probe potential in Fig. 4 is given relative to that of the burner, which served as the reference electrode.

Examination of Fig. 4 shows that the ion current saturates at about $\varphi = -0.5$ V and the electron current at $\varphi = 2$ V. For comparison with the theoretical results, we must determine and nondimensionalize the corresponding values of the potentials relative to the plasma potential.

The plasma potential may be estimated as follows. From Figs. 1 and 2, for $\tau = 1$, when the probe potential is equal to the plasma potential $\psi_p = 0$, the dimensionless positive and negative currents are equal and constitute half of the saturation currents. The saturation currents themselves, as follows from (5.1), vary with the corresponding diffusion coefficients. From this, we find that the total dimensional current J_0 at the plasma potential is given in terms of the ion saturation current J_+^* by

$$J_0 = 0{,}5 J_+^* (1 - 1/\beta).$$

Using the data of [16], we may estimate that for T = 2370 K in an air-acetylene flame, the electron diffusion coefficient is $D_- = 240$ cm^2/sec. Thus, the electron saturation current should exceed the ion current by a factor of about 46. However, as seen in Fig. 4, the actual excess is only a factor of 7.

The ratio of electrical to ion saturation current decreases for $\tau < 1$. Because the temperature at the probe surface is always lower than that of the undisturbed flame, it is possible that the electron temperature differs from that of the much heavier positive ions at the

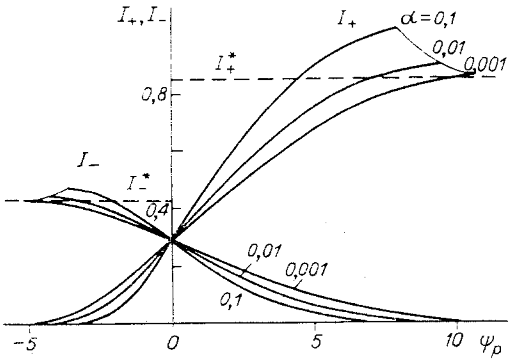

Fig. 3

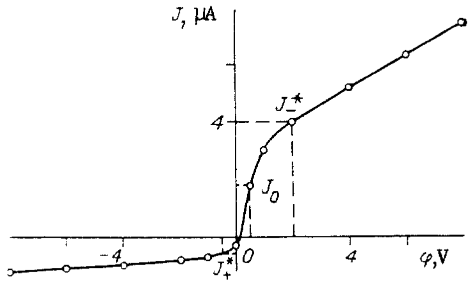

Fig. 4

probe surface. The current ratio also decreases under the assumption that negative charge is carried partly by ions, whose diffusion coefficient is considerably lower than that of electrons. Previous work [8] has shown that the fraction of negative ions may be substantial at colder probe surfaces.

Based on Fig. 4, taking an effective value of $\beta = 0.15$ and assuming that the ion current at −0.5 V corresponds to the theoretical saturation current (neglecting the effect of finite α), we find that at the plasma potential the probe current is 1.7 μA while the plasma potential is about 0.5 V. The dimensionless probe potentials at which the currents saturate are $\psi_p \simeq 5$ for ions and $\psi_p \simeq -7.5$ for electrons, somewhat lower than given in Sec. 4.

The results of this work may be useful in calculating voltage-current characteristics of a cylindrical probe for $Re_e \sim 1$.

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